Iteration of Mandelbrot cascades in a random environment

The original Mandelbrot cascades operate on a homogeneous tree: starting from a random variable W of expectation 1, the Mandelbrot construction provides a martingale which, when non-degenerate, converges to a new variable Y of expectation 1. The distribution of Y depends only on the distribution of W, so this process defines a mapping T from probability distributions to probability distributions. Starting from a suitable distribution, this mapping can be iterated. This gives rise to a central limit theorem. When dealing in a random environment, new phenomena occur. Depending on the environment, there are two regimes. In the so-called conservative case, there is a central limit theorem which may be non-standard (i.e., the limit law needs not be normal). In the non-conservative case, the map T has a unique fixed point which is also a fixed point of a non-linear smoothing transformation.