

## Abstract

For a tuple  $A = (A_1, A_2, \dots, A_n)$  of elements in a unital Banach algebra  $\mathcal{B}$ , its *projective joint spectrum*  $P(A)$  is the collection of  $z \in \mathbf{C}^n$  such that the multiparameter pencil  $A(z) = z_1 A_1 + z_2 A_2 + \dots + z_n A_n$  is not invertible. If  $\mathcal{B}$  is the group  $C^*$ -algebra for a discrete group  $G$  generated by  $A_1, A_2, \dots, A_n$  with respect to a representation  $\rho$ , then  $P(A)$  is an invariant of (weak) equivalence for  $\rho$ . This talk presents some recent work on the joint spectrum  $P(R)$  of  $R = (1, a, t)$  for the infinite dihedral group  $D_\infty = \langle a, t \mid a^2 = t^2 = 1 \rangle$  with respect to the left regular representation. Results include a description of the joint spectrum, a formula for the Fuglede-Kadison determinant of the pencil  $R(z) = 1 + z_1 a + z_2 t$ , the first singular homology group of the joint resolvent set  $P^c(R)$ , and dynamical properties of the joint spectrum. These results give new insight into some earlier studies on groups of intermediate growth. This is a joint work with R. Grigorchuk.