Remarks on the approximate Lagrangian controllability of Euler Equation

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Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with a regular boundary $\partial\Omega$, and let Γ be a part of $\partial\Omega$ with nonempty relative interior. Assume that a subdomain $\omega \subset\subset \Omega$ is given such that its boundary $\gamma := \partial\omega$ is a Jordan curve and let us denote by $\mathbf{n}$ the exterior normal to the boundary of $\Omega \setminus \overline{\omega}$. The question we address in this talk is the following: given a function h defined on γ, can one find a function v defined on $\partial\Omega$ having its support $\mathrm{supp}(v) \subset \Gamma$, and such that the solution Ψ of

$$\Delta\Psi = 0 \quad \text{in } \Omega, \qquad \frac{\partial\Psi}{\partial\mathbf{n}} = v \quad \text{on } \partial\Omega, \tag{1}$$

satsifies

$$\frac{\partial\Psi}{\partial\mathbf{n}} = h \quad \text{on } \gamma \text{ ?} \tag{2}$$

The motivation of this question lies in its application to the approximate Lagrangian control of Euler equation.

The talk will also address the following ill-posed problem: let Ω be the rectangular domain $\Omega := (0,\pi) \times (0,\ell) \subset \mathbb{R}^2$ for some $\ell > 0$ and set $\Gamma_0 := [0,\pi] \times \{0\}$ and $\Gamma := \{0\} \times [0,\ell] \cup \{\pi\} \times [0,\ell]$. We give necessary and sufficient conditions on the Cauchy data f_0, g_0 so that there exists $u \in H^1(\Omega)$ satisfying $\Delta u = 0$ in Ω and

$$\frac{\partial u}{\partial\mathbf{n}} = 0 \quad \text{on } \Gamma, \qquad u = f_0 \quad \text{on } \Gamma_0 \quad \text{and} \quad \frac{\partial u}{\partial\mathbf{n}} = g_0 \quad \text{on } \Gamma_0. \tag{3}$$