复旦大学数学科学学院

数学综合报告会

报告题目：Mini course on numerical integration 1，2，3

报告人：Peter Mathé（Weierstrass Institute for Applied Analysis and Stochastics）

时间：11 月 20 日，11 月 27 日，12 月 4 日（13:00 PM - 15:15 PM ）

地点：光华西辅楼 203

摘要：In this mini-course we shall highlight different aspects of numerical integration. Theoretical investigations are briefly accompanied with numerical experiments such that the students can later apply the theoretical results for specific applications.

1. Quadrature formulas

The theory of numerical integration in one dimension has a long history. We shall review the basic concepts related to such quadrature formulas, as these are interpolatory formulas, and the degree of exactness of such formulas. The highlight will be the fundamental Theorem for Gauss quadratures.

2. Cubature formulas

Numerical integration in higher dimension, specifically, when integration is on tensor product space, also known as cubature, uses the results from the one dimensional theory. However, in order to get efficient cubature formulas one may consider formulas based on 'sparse grids'.

3. Monte Carlo methods

In higher dimensions classical cubature formulas often suffer from the 'curse of dimension'. In order to circumvent this one may change the numerical setting and use 'randomized' cubature formulas, which constitute the basis of Monte Carlo methods. This is a versatile means which provides us with 'statistical error' bounds that do not depend on the spatial dimension.

4. Quasi-Monte Carlo methods

In order to avoid the randomness in the results of the numerical integration one may sometimes replace the Monte Carlo methods by their deterministic counterpart, known as quasi-Monte Carlo methods. We review the fundamental concepts, in particular the role of the discrepancy, the corresponding low discrepancy sequences, and the Koksma--Hlawka Inequality, which provides the basis for errorestimates.

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