



# 复旦大学数学科学学院 数学综合报告会

报告题目：**Sharp spectral transitions and growth of eigenfunctions of Laplacians on Riemannian manifolds**

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报告时间：2017-06-19 星期一 15:00-16:00

报告地点：光华东主楼 1801

**摘要：** We study the eigenvalues or singular continuous spectrum of the free Laplacian embedded in the essential spectrum (absolutely continuous spectrum) on either asymptotically flat or asymptotically hyperbolic manifolds. The essential spectrum is  $[\frac{c^2}{4}, \infty)$  if  $\Delta r \rightarrow c$  as  $r$  goes to infinity, where  $r(x)$  is the distance function. Kumura proved that there are no eigenvalues embedded in the essential spectrum  $\sigma_{\text{ess}}(-\Delta) = \left[ \frac{1}{4}(n-1)^2, \infty \right)$  of Laplacians on asymptotically hyperbolic manifolds, where asymptotic hyperbolicity is characterized by the radial curvature, i.e.,  $K_{\text{rad}} = -1 + o(r^{-1})$ . He also constructed a manifold for which an eigenvalue  $\frac{(n-1)^2}{4} + 1$  is embedded into its essential spectrum  $[\frac{(n-1)^2}{4}, \infty)$  with the radial curvature  $K_{\text{rad}}(r) = -1 + o(r^{-1})$ .

The first part of the talk, based on a joint work with S. Jitomirskaya, is devoted to construction of manifolds with embedded eigenvalues and singular continuous spectrum. Given any finite (countable) positive energies  $\{\lambda_n\} \subset [\frac{K_0}{4}(n-1)^2, \infty)$ , we construct Riemannian manifolds with the decay of order  $K_{\text{rad}} + K_0 = o(r^{-1})$  with  $K_0 \geq 0$  ( $K_{\text{rad}} + K_0 = \frac{C(r)}{r}$ ), where  $C(r) \geq 0$  and  $C(r) \rightarrow \infty$  arbitrarily slowly) such that the eigenvalues  $\{\lambda_n\}$  are embedded in the essential spectrum  $\sigma_{\text{ess}}(-\Delta) = \left[ \frac{K_0}{4}(n-1)^2, \infty \right)$ .

We also construct Riemannian manifolds with the decay of order  $K_{\text{rad}} + K_0 = \frac{C(r)}{r}$ , where  $C(r) \geq 0$  and  $C(r) \rightarrow \infty$  arbitrarily slowly such that there is singular continuous spectrum embedded in the essential spectrum  $\sigma_{\text{ess}}(-\Delta) = \left[ \frac{K_0}{4}(n-1)^2, \infty \right)$ .

In the second part, I discuss criteria for the absence of eigenvalues embedded into essential spectrum in terms of the asymptotic behavior of  $\Delta r$ . Under a weaker convexity condition.

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