Title:Variations of Kähler-Einstein metrics on strongly pseudoconvex domains

Abstract: Let D be a smooth domain in $\mathbb{C}^{n+m}$ which satisfies that for each $s \in \pi(D)$, the fiber $D_s := \pi^{-1}(s)$ is a bounded strongly pseudoconvex domain with smooth boundary. A celebrated theorem of Cheng and Yau implies that on each fiber D_s there exists a unique complete Kähler metric $h_{\alpha\bar{\beta}}(z,s) := h^s_{\alpha\bar{\beta}}(z)$ which satisfies:

$$-(n+1)h_{\alpha\bar{\beta}}(z,s) = -\frac{\partial^2}{\partial z^\alpha z^{\bar{\beta}}} \log\det\left(h_{\gamma\bar{\delta}}(z,s)\right)_{1\le\gamma,\delta\le n},$$

namely, the Ricci curvature is negative constant $-(n+1)$. This unique complete Kähler metric is called *Kähler-Einstein metric*. Hence on each fiber D_s,

$$\frac{1}{n+1}\log\det\left(h_{\gamma\bar{\delta}}(z,s)\right)_{1\le\gamma,\delta\le n}$$

is a potential function of the Kähler-Einstein metric $h_{\alpha\bar{\beta}}$. Denote it by $h(z,s)$. Consider the function $h(z,s)$ on D. It is an immediate consequence of the Kähler-Einstein condition is that the restriction of H to each fiber D_s is strictly plurisubharmonic. But it is not obvious that it is also plurisubharmonic or strictly plurisubharmonic in base direction (s-direction).

In this talk, we discuss the plurisubharmonicity of $h(z,s)$ with respect to s-variable when the total space D is pseudoconvex. We also discuss the local triviality of the family of bounded strongly pseudoconvex domains.