



复旦大学数学科学学院 数学综合报告会

报告题目: **Logarithmic Corrections in Fisher-KPP Problems for the Porous Medium Equation**

报告人: Dr. Maolin Zhou

(University of New England, Australia)

报告时间: 2017-9-20 周三 10:30--11:30

报告地点: 光华东主楼 1801

摘要:

ABSTRACT. We consider the large time behaviour of solutions to the porous medium equation with a Fisher-KPP type reaction term and nonnegative, compactly supported initial function in $L^\infty(\mathbb{R}^N) \setminus \{0\}$:

$$(*) \quad u_t = \Delta u^m + u - u^2 \quad \text{in } Q := \mathbb{R}^N \times \mathbb{R}_+, \quad u(\cdot, 0) = u_0 \quad \text{in } \mathbb{R}^N,$$

It is well known that the spatial support of the solution $u(\cdot, t)$ to this problem remains bounded for all time $t > 0$. In spatial dimension one it is known that there is a minimal speed $c_* > 0$ for which the equation admits a traveling wave solution Φ_{c_*} with a finite front, and this traveling wave solution is asymptotically stable in the sense that if the initial function $u_0 \in L^\infty(\mathbb{R})$ satisfies $\liminf_{x \rightarrow -\infty} u_0(x) > 0$ and $u_0(x) = 0$ for all large x , then $\lim_{t \rightarrow \infty} \{\sup_{x \in \mathbb{R}} |u(x, t) - \Phi_{c_*}(x - c_*t - x_0)|\} = 0$ for some $x_0 \in \mathbb{R}$. In dimension one we obtain an analogous stability result for the case of compactly supported initial data, not necessarily symmetric. In higher dimensions we show that Φ_{c_*} is still attractive, albeit that a logarithmic shifting occurs. More precisely, if the initial function in (*) is additionally assumed to be radially symmetric, then there exists a second constant $c^* > 0$ independent of the dimension N and the initial function u_0 , such that

$$\lim_{t \rightarrow \infty} \left\{ \sup_{x \in \mathbb{R}^N} |u(x, t) - \Phi_{c_*}(|x| - c_*t + (N-1)c^* \log t - r_0)| \right\} = 0$$

for some $r_0 \in \mathbb{R}$ (depending on u_0). If the initial function is not radially symmetric, then there exist $r_1, r_2 \in \mathbb{R}$ such that the boundary of the spatial support of the solution $u(\cdot, t)$ for all large time t is contained in the spherical shell $\{x \in \mathbb{R}^N : r_1 \leq |x| - c_*t + (N-1)c^* \log t \leq r_2\}$, and for any $c \in (0, c^*)$, $\lim_{t \rightarrow \infty} u(x, t) = 1$ uniformly in $\{|x| \leq c_*t - (N-1)c \log t\}$. It is a joint work with Yihong Du in University of New England and Fernando Quirós in Universidad Autónoma de Madrid.