LOGARITHMIC CORRECTIONS IN FISHER-KPP PROBLEMS FOR THE POROUS MEDIUM EQUATION

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Abstract. We consider the large time behaviour of solutions to the porous medium equation with a Fisher-KPP type reaction term and nonnegative, compactly supported initial function in $L^\infty(\mathbb{R}^N) \setminus \{0\}$:

$$(*) \quad u_t = \Delta u^m + u - u^2 \quad \text{in } Q := \mathbb{R}^N \times \mathbb{R}_+, \qquad u(\cdot, 0) = u_0 \quad \text{in } \mathbb{R}^N,$$

It is well known that the spatial support of the solution $u(\cdot, t)$ to this problem remains bounded for all time $t > 0$. In spatial dimension one it is known that there is a minimal speed $c_* > 0$ for which the equation admits a traveling wave solution Φ_{c_*} with a finite front, and this traveling wave solution is asymptotically stable in the sense that if the initial function $u_0 \in L^\infty(\mathbb{R})$ satisfies $\liminf_{x\to-\infty} u_0(x) > 0$ and $u_0(x) = 0$ for all large x, then $\lim_{t\to\infty} \{\sup_{x\in\mathbb{R}} |u(x, t) - \Phi_{c_*}(x - c_* t - x_0)|\} = 0$ for some $x_0 \in \mathbb{R}$. In dimension one we obtain an analogous stability result for the case of compactly supported initial data, not necessarily symmetric. In higher dimensions we show that Φ_{c_*} is still attractive, albeit that a logarithmic shifting occurs. More precisely, if the initial function in $(*)$ is additionally assumed to be radially symmetric, then there exists a second constant $c^* > 0$ independent of the dimension N and the initial function u_0, such that

$$\lim_{t\to\infty} \left\{ \sup_{x\in\mathbb{R}^N} |u(x, t) - \Phi_{c_*}(|x| - c_* t + (N-1)c^* \log t - r_0)| \right\} = 0$$

for some $r_0 \in \mathbb{R}$ (depending on u_0). If the initial function is not radially symmetric, then there exist $r_1, r_2 \in \mathbb{R}$ such that the boundary of the spatial support of the solution $u(\cdot, t)$ for all large time t is contained in the spherical shell $\{x \in \mathbb{R}^N : r_1 \le |x| - c_* t + (N-1)c^* \log t \le r_2\}$, and for any $c \in (0, c^*)$, $\lim_{t\to\infty} u(x, t) = 1$ uniformly in $\big\{|x| \le c_* t - (N-1)c \log t\big\}$. It is a joint work with Yihong Du in University of New England and Fernando Quirós in Universidad Autónoma de Madrid.