

On the Problem of Graph Isomorphism

Wen-Xue Du

School of Mathematical Science, Anhui University, Hefei, 230039, China

E-mail: wenxuedu@gmail.com

Abstract

Let G and H be two simple graphs. A bijective map $\phi : V(G) \rightarrow V(H)$ is called an *isomorphism* between G and H if $\phi(v_i)\phi(v_j) \in E(H) \Leftrightarrow v_iv_j \in E(G)$ for any two vertices v_i and v_j of G , and G and H are said to be isomorphic if there exists such a map. In the case that $G = H$, we say ϕ an automorphism of G and denote the group consisting of automorphisms of G by $\text{Aut } G$. One of key steps in determining whether or not G is isomorphic to H is to characterize the structure of the action of $\text{Aut } G$ on $V(G)$. By means of bipartite graphs $\{[\Pi_u^*, \Pi_v^*] : u, v \in V(G)\}$ and other combinatorial constructions alike, we can reduce the problem of determining the partition Π_G^* , composed of orbits of $\text{Aut } G$, to that of working out the family of partitions $\{\Pi_v^* : v \in V(G)\}$, where Π_v^* is comprised of orbits of the stabilizer $(\text{Aut } G)_v$.

On the other hand, we have a permutation group $\text{Aut } U$ for each subspace $U \subseteq \mathbb{R}^n$, which is composed of those permutations in S_n for each of which U is invariant. Then $\text{Aut } G = \bigcap_{\lambda \in \text{spec } \mathbf{A}(G)} \text{Aut } V_\lambda$, where V_λ is the eigenspace of $\mathbf{A}(G)$ corresponding to λ . Accordingly, we can obtain the structure of $\text{Aut } G$ action by means of that of $\text{Aut } V_\lambda$ action, $\lambda \in \text{spec } \mathbf{A}(G)$. The latter can be obtained by analyzing the division of V_λ by subspaces $\{\text{proj}[V_\lambda](\mathbf{e}_v)^\perp : v \in V(G)\}$, where $\text{proj}[V_\lambda](\mathbf{e}_v)^\perp$ stands for the orthogonal complement of $\text{proj}[V_\lambda](\mathbf{e}_v)$ in V_λ . As a application, we devise a new algorithm solving the Graph Isomorphism problem in time $n^{O(\log n)}$, which is equal to $e^{O(\log^2 n)}$.