

Critical edge behavior in the perturbed Laguerre unitary ensemble and the Painlevé V transcendent

Chen Min,

Joint work with prof. Fan Engui and prof. Chen Yang

Abstract

We study the perturbed Laguerre unitary ensemble described by the weight function of $w(x, t) = (x + t)^\lambda x^\alpha e^{-x}$, $x \geq 0$, $t > 0$, $\alpha > 0$, $\alpha + \lambda + 1 > 0$. In this paper, we adopt Deift-Zhou nonlinear steepest descent approach to study the limit of the eigenvalue correlation kernel. At the hard edge, under the double scaling $s = 4nt$, $n \rightarrow \infty$, $t \rightarrow 0$, such that s is positive and finite, the limiting kernel described by the φ -functions that are related to a new third-order nonlinear differential equation, which can be translated to a particular Painlevé V transcendent (shorted as P_V) via a simple transformation. Moreover, this P_V transcendent is equivalent to the general Painlevé III transcendent. For large s , the P_V kernel reduces to the Bessel kernel $\mathbf{J}_{\alpha+\lambda}$. For small s , the P_V kernel degenerates to another Bessel kernel \mathbf{J}_α .